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GROUP {1, −1, *i*, −*i*} CORDIAL LABELING OF PRODUCT RELATED GRAPHS

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ABSTRACT

Let G be a (p,q) graph and A be a group. Let $f : V(G) \rightarrow A$ be a function. The order of $u \in A$ is the least positive integer n such that $u^n = e$. We denote the order of u by $o(u)$. For each edge uv assign the label 1 if $(o(u), o(v)) = 1$ or 0 otherwise. f is called a group A Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with $n (n = 0, 1)$. A graph which admits a group A Cordial labeling is called a group A Cordial graph. In this paper we define group {1,−1, *i*,−*i*} Cordial graphs and prove that Hypercube $Q_n = Q_{n-1} \times K_2$, Book $B_n = S_n \times K_2$, n-sided prism $Pr_n = C_n \times K_2$ and $P_n \times K_3$ are all group {1,−1, *i*,−*i*} Cordial for all n.

INTRODUCTION

Graphs considered here are finite, undirected and simple. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$. Cahit[4] introduced the concept of Cordial labeling. Motivated by this, we defined group A cordial labeling and investigated some of its properties. We also defined group {1,−1, *i*,−*i*} cordial labeling and discussed that labeling for some standard graphs[2, 3]. In this paper we discuss the labeling for some product related graphs. Terms not defined here are used in the sense of Harary[6] and Gallian[5].

The greatest common divisor of two integers m and n is denoted by (m, n) and m and n are said to be *relatively prime* if $(m, n) = 1$. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x.

A *path* is an alternating sequence of vertices and edges, $v_1, e_1, v_2, e_2, \ldots, e_{n-1}, v_n$, which are distinct, such that e_i is an edge joining v_i and v_{i+1} for $1 \leq i \leq n - 1$. A path on n vertices is

denoted by P_n. A path $v_1, e_1, v_2, e_2, ..., e_{n-1}, v_n, e_n, v_1$ is called a cycle and a cycle on n vertices is denoted by C_n. A graph $G = (V,E)$ is called a bipartite graph if $V = V_1 \cup V_2$ and every edge of G joins a vertex of V_1 to a vertex of V_2. If $|V_1| = m$, $|V_2| = n$ and if every vertex of V_1 is adjacent to every vertex of V_2, then G is called a complete bipartite graph and is denoted by $K_{m,n}$. $K_{1,n}$ is called a star. If G is a graph on n vertices in which every vertex is adjacent to every other vertex, then G is called a complete graph and is denoted by K_n.

The Cartesian product $G \times H$ of two graphs G and H is a graph whose vertex set is the Cartesian product $V(G) \times V(H)$ and two vertices (u, u') and (v, v') are adjacent in $G \times H$ if and only if either $u = v$ and $u'v'$ is an edge in H or $u' = v'$ and uv is an edge in G. We need the following theorem.

Theorem – 1:[7] *Given a collection of n distinct objects, the number of ways of selecting an odd number of objects is equal to the number of ways of selecting an even number of objects.*

1.1 Group {1,−1, *i,−i*} Cordial Graphs

Definition – 1: Let G be a (p,q)graph and consider the group A = {1,−1, *i,−i*} with multiplication. Let $f : V(G) \to A$ be a funtion. For each edge uv assign the label 1 if $(o(u), o(v)) = 1$ or 0 otherwise. f is called a group {1,−1, *i,−i*} Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with $n(n = 0, 1)$. A graph which admits a group {1,−1, *i,−i*} Cordial labeling is called a group {1,−1, *i,−i*} Cordial graph.

Example – 1: A simple example of a group {1,−1, *i,−i*} Cordial graph is given in Fig – 1.

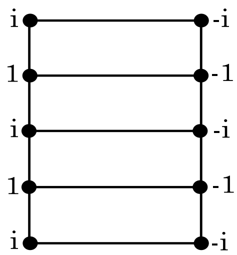
We now investigate the group {1,−1, *i,−i*} Cordial labeling of product of some (p, q) graphs.

Definition – 2: The $n - Cube$ Q_n is the graph whose vertex set is the set of all $n-$ dimensional boolean vectors ($n-$tuples), two vertices being joined if they differ in exactly one coordinate. It can be defined recursively by $Q_1 = K_2$ and $Q_n = K_2 \times Q_{n-1}$.

Arumugam and Kala[1] introduced the following notation:

Notation – 1: By (0), we denote the boolean vector with all coordinates 0. If $1 \leq i_1 < i_2 < \cdots < i_k \leq n$, we denote by $(i_1, i_2, \ldots, i_k)$ the $n-$ tuple having 1 in the coordinates $i_1, i_2, \ldots, i_k$ and 0 elsewhere.

Figure 1:



Theorem – 2: *Hypercube Q_n is group {1,−1, i,−i} Cordial for every n.*

Proof: Q_n has 2^n vertices and $n.2^{n-1}$ edges. Each vertex label should occur 2^{n-2} times and each edge label $n.2^{n-2}$ times. For $1 \leq n \leq 3$, a group {1,−1, *i,−i*} Cordial labeling is given in Table 1. Suppose $n \geq 4$. Choose $\binom{n}{0}$

Table 1:

n	(0)	(1)	(2)	(3)	(1,2)	(1,3)	(2,3)	(1,2,3)
1	1	-1						
2	1	-1	*i*	-*i*				
3	1	-1	-1	*i*	1	*I*	-*i*	-*i*

vertices having no coordinate of the n-tuple as 1, $\binom{n}{2}$ vertices having 2 coordinates of the n-tuple as 1 and so on. If n is odd, number of vertices chosen is $\binom{n}{0} + \binom{n}{2} + \cdots + \binom{n}{n-1}$ and if n is even, number of vertices chosen is $\binom{n}{0} + \binom{n}{2} + \cdots + \binom{n}{n}$. By Theorem 1, both are equal to 2^{n-1}. Of these 2^{n-1} vertices, choose 2^{n-2} vertices arbitarily and give label 1. Each vertex is incident with n distinct edges and so $n.2^{n-2}$ edges get label 1 . Of the selected 2^{n-1} vertices, label the remaining 2^{n-2} vertices with label −1. Label the remaining vertices arbitarily so that 2^{n-2} of them get label i and 2^{n-2} of them get label $-i$. This is a group $\{1,-1, i,-i\}$ Cordial labeling of Q_n.

Example – 2: *A group* $\{1,-1, i,-i\}$ *Cordial labeling of* Q_4 *is given below.*

Define f by,

$f((0)) = f((1, 2)) = f((1, 3)) = f((1, 4)) = 1$

$f((2, 3)) = f((2, 4)) = f((3, 4)) = f((2, 3, 4)) = -1$

$f((1)) = f((2)) = f((3)) = f((4)) = i$

$f((1, 2, 3)) = f((1, 2, 4)) = f((1, 3, 4)) = f((2, 3, 4)) = -i$

Theorem – 3: $P_n \times K_3$ *is group* $\{1,-1, i,-i\}$ *Cordial for all n.*

Proof: Let the vertices of the 3 copies of P_n in $P_n \times K_3$ be labeled as $u_1,u_2,......,u_n,v_1,v_2,......,v_n$ and $w_1,w_2,.....,w_n$ respectively. Number of vertices in $P_n \times K_3$ is 3_n and number of edges is $6n - 3$.

Case(i): $3n \equiv 0 \ (mod\ 4)$

Let $3n = 4r$, ($r \in Z+$). Define a labeling f as follows:

If r is odd , label the vertices $u_2,w_3,u_4,w_5,u_6,w_7,\cdots,u_{r-1},w_r,w_2$ by 1 and if r is even, label the vertices $u_2,w_3,u_4, \cdots, w_{r-1},u_r,w_2$ by 1. Label the remaining vertices arbitarily so that r vertices get label −1, r vertices get label i and r vertices get label $-i$. Number of edges with label 1 is $4r - 2$.

Case(ii): $3n \equiv 1 \ (mod 4)$

Let $3n = 4r + 1$, ($r \in Z+$). Define a labeling f as follows:

If r is odd , label the vertices $u_2,w_3,u_4,w_5,\cdots,u_{r-1},w_r,v_2$ by 1 and if r is even, label the vertices $u_2,w_3,u_4, \cdots, w_{r-1},u_r,v_2$ by 1. Label the remaining vertices arbitarily so that $r+1$ vertices get label −1, r vertices get label i and r vertices get label $-i$. Number of edges with label 1 is $4r - 1$.

Case(iii): $3n \equiv 2\ (mod4)$

Let $3n = 4r + 2$, ($r \in Z+$). For $r = 1$, $n = 2$. The function f defined by $f(u1)=f(v1) = 1$, $f(u2)=f(v2) = -1$, $f(w1) = i$ and $f(w2) = -i$ is a group $\{1,-1, i,-i\}$ Cordial labeling of $P_2 \times K_3$. Suppose $r \geq 2$. Define a labeling f as follows:

If $r+1$ is odd , label the vertices $u_2, w_3, u_4, \cdots, w_{r+1}$ with 1 and if $r+1$ is even, label the vertices $u_2, w_3, u_4, \cdots, w_r, u_{r+1}$ with 1. Label the remaining vertices arbitarily so that $r + 1$ vertices get label -1, $r + 1$ vertices get label i and r vertices get label $-i$. Number of edges with label 1 is $4r$.

Case(iv): $3n \equiv 3\ (mod4)$

Let $3n = 4r + 3$, ($r \in Z+$). Define a labeling f as follows:

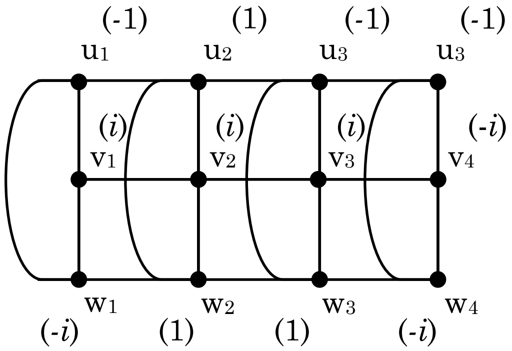
If r is odd, label the vertices $u_2, v_2, w_2, u_4, w_5, u_6, w_7, \cdots, u_{r+1}$ with 1 and if r is even, label the vertices $u_2, v_2, w_2, u_4, w_5, u_6, w_7, \cdots, u_{r+1}, w_{r+1}$ with 1. Label the remaining vertices arbitarily so that $r + 1$ vertices get label -1, $r + 1$ vertices get label i and r vertices get label $-i$. Number of edges with label 1 is $4r + 1$.

Table – 2: *Shows that in all 4 Cases, the function f defined is a Group {1,−1, i,−i} Cordial Labeling.*

$3n$	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
$4r$	r	r	r	r	$4r - 1$	$4r - 2$
$4r + 1$	r	$r + 1$	r	r	$4r$	$4r - 1$
$4r + 2 (r = 1)$	$r + 1$	$r + 1$	r	r	$4r$	$4r + 1$
$4r + 2 (r \geq 2)$	r	$r + 1$	r	r	$4r + 1$	$4r$
$4r + 3$	$r + 1$	$r + 1$	$r + 1$	r	$4r + 2$	$4r + 1$

Example – 3: *An illustration for $P_4 \times K_3$ is given in Fig 2.*

Figure 2



Definition – 3: *The book B_n is the graph $S_n \times P_2$ where S_n is the star with n edges.*

Theorem – 4: The Book B_n is group $\{1,-1, i,-i\}$ cordial for all n.

Proof: Let $V(B_n) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_n) = \{uv, uu_i, vv_i, u_iv_i : 1 \leq i \leq n\}$. Clearly order of B_n is $2n+2$ and the size is $3n+1$. Define a map f from $V(B_n)$ to the group $\{1,-1, i,-i\}$ as follows:

Case(1): n is even.

$f(u) = 1$, $f(v) = -1$

$f(u_j) = 1$, $1 \le j \le \left(\frac{n}{2} - 1\right)$

$f(u_{\left(\frac{n}{2}\right)+j-1}) = -1$, $1 \le j \le \left(\frac{n}{2}\right)$

$f(un) = -i$

$f(v_j) = i$, $1 \le j \le \left(\frac{n}{2}\right) + 1$

$f(v_{j+\left(\frac{n}{2}\right)+1}) = -i$, $1 \le j \le \left(\frac{n}{2}\right) - 1$.

Case(2): n is odd.

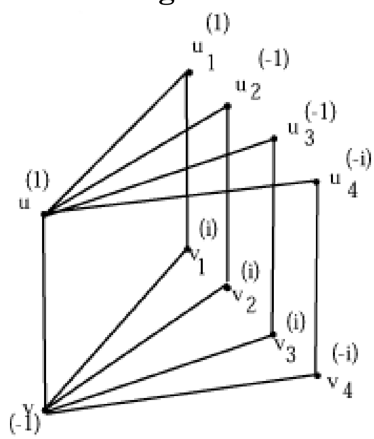
Assign the label to the vertices u, v, u_i, v_i ($1 \le i \le n-1$) as in Case(1). Finally assign the label $-i$ and 1 respectively to the vertices u_n, v_n.

Table – 3 establish that f is a group $\{1,-1, i,-i\}$ cordial labeling.

Parity of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
even	$\frac{n}{2}$	$\frac{n}{2}+1$	$\frac{n}{2}+1$	$\frac{n}{2}$	$\frac{3n+2}{2}$	$\frac{3n}{2}$
odd	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$

Example – 4: *An illustration for B_4 is given in Fig – 3.*

Figure 3



Definition – 4: *An n-sided prism Pr_n is a planar graph having 2 faces viz, an inner face and outer face with n sides and every other face is a 4-cycle. In other words, it is $C_n \times K_2$.*

Theorem – 5: *An n-sided prism Prn is group* $\{1,-1, i,-i\}$ *cordial for every n.*

Proof: Let the vertices of the inner face be labeled as $u_1, u_2, \ldots, u_n$ in order and the vertices of the outer face are labeled as $v_1, v_2, \ldots, v_n$ in order so that so that u_i is adjacent to v_i for every i, $1 \le i \le$ n. Number of vertices in Pr_n is $2n$ and number of edges is $3n$.

Case(1): n is even.

Let $n = 2k$, $k \geq 2$, $k \in Z$. Define a function f on $V(G)$ as follows:

$f(u_1) = f(u_3) = f(u_5) = \cdots = f(u_{2k-1}) = 1$

$f(u_2) = f(u_4) = f(u_6) = \cdots = f(u_{2k}) = -1$

$f(v_1) = f(v_3) = f(v_5) = \cdots = f(v_{2k-1}) = i$

$f(v_2) = f(v_4) = f(v_6) = \cdots = f(v_{2k}) = -i$

Case(2): n is odd.

Let $n = 2k + 1$, $k \geq 1$, $k \in Z$. Define a function f on $V(G)$ as follows:

$f(u_1) = f(u_3) = f(u_5) = \cdots = f(u_{2k-1}) = f(v_1) = 1$

$f(u_2) = f(u_4) = f(u_6) = \cdots = f(u_{2k}) = f(v_2) = -1$

$f(u_{2k+1}) = f(v_3) = f(v_4) = \cdots = f(v_{k+1}) = i$

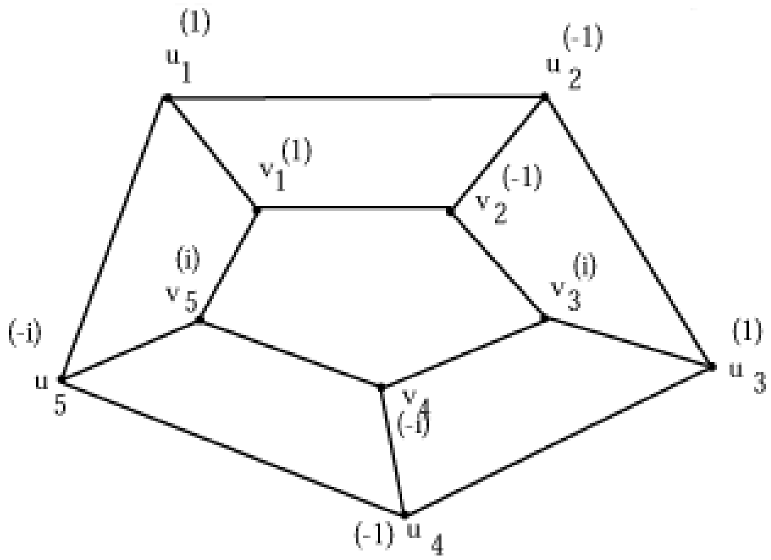
$f(v_{k+2}) = f(v_{k+3}) = f(v_6) = \cdots = f(v_{2k+1}) = -i$

Table 4 shows that f is a group $\{1,-1, i,-i\}$ cordial labeling of Pr_n.

Parity of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
even	k	k	k	k	$3k$	$3k$
odd	$k + 1$	$k + 1$	k	k	$3k + 1$	$3k + 2$

Example – 5: *An illustration for Pr_5 is given in Fig 4.*

Figure 4



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